**9020 Assignment 1**

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**Problem 1**

Symmetric:

There is a truth table:

|  |  |  |
| --- | --- | --- |
|  |  |  |
| T | T | T |
| F | F | T |

Therefore, ≡, satisfies symmetric on

Reflexive:

There is a truth table:

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| T | T | T | T |
| F | F | T | T |
| T | F | F | F |
| F | T | F | F |

Therefore, ≡, satisfies reflexive on

Transitive

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
| T | T | T | T | T | T |
| F | F | F | T | T | T |
| T | F | F | F | T | F |
| T | T | F | T | F | F |
| T | F | T | F | F | T |
| F | F | T | T | F | F |
| F | T | F | F | F | T |
| F | T | T | F | T | F |

Therefore, ≡, satisfies transitive on

Therefore, ≡, is an equivalence relation on

(i)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
| T | T | T | F | F | T |
| F | F | T | T | T | T |
| T | F | F | F | T | F |
| F | T | F | T | F | F |

According to truth table:

When is truth, is truth.

(ii)

When and

There is truth table:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |
| T | T | T | T | T | T | T |
| T | T | F | F | F | F | T |
| F | F | T | T | F | F | T |
| F | F | F | F | F | F | T |

Thus,

When, and ,

(iii)

When and

There is truth table:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |
| T | T | T | T | T | T | T |
| T | T | F | F | T | T | T |
| F | F | T | T | T | T | T |
| F | F | F | F | F | F | T |

Thus,

When, and ,

Support and

Identity:

Complementation:

Commutative:

Therefore,

Therefore,

Associative:

Therefore,

Therefore,

Distributive:

Therefore,

Therefore,

Thus, is a Boolean Algebra.

**Problem 3**

1. Q

The Petersen graph does not contain a subdivision of .

This is because:

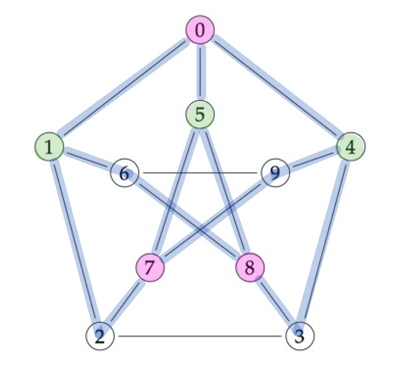
图片包含 室内, 浅色, 电子产品

描述已自动生成

This is , every vertex has 4 degree.

However, in the given graph, every vertex has 3 degree.

Therefore, the given graph does not contain a subdivision of .

****

We can find the ’s graph isomorphism in the Petersen graph

Therefore, the Petersen graph contains a subdivision of .

**Problem 3**

According to given course selection conditions：

(i)

Let’s:

Vertex represents course:

①: Defence against the Dark Arts

②: Potions

③: Herbology

④: Transfiguration

⑤: Charms

There are five vertices.

Every edge connects the course that does not clashes.

(ii)

According to the clique number of the graph () to represent the maximum number of classes he can take.

We can draw a graph depended on course selection conditions



Every clique is the possibility of course selection. The maximum clique has two vertices.

Therefore, the number of maximum classes he can take is 2.

**Problem 4**

When

(one empty tree)

Therefore,

When

1𝑛+1∙𝑛 (𝑛≥1)

According to assignment2, we have

In a full binary tree (a non-empty binary tree where every node has either two non-empty children or two empty children).

Therefore, in a full binary tree

Thus, a full binary tree must have an odd number of nodes.

struct leaves(T \* root){

if (root == NULL)

{

return 0

}

if (root->left == NULL && root->right == NULL)

{

return 1;

}

return leaves(root->left) + leaves(root->right);

}

struct internal(T \* root){

if (root == NULL)

{

return -1

}

if (root->left == NULL && root->right == NULL)

{

return 0;

}

return internal(root->left) + internal(root->right) + 1;

}

Firstly, starting to consider from the binary tree’s top to bottom:

Every internal node () has 2 branches.

Every leave () has 0 branch.

Every not internal node () has one branch.

Secondly, starting to consider from the binary tree’s bottom to top:

Support a binary tree has N (that include , , ) nodes, every node owns one branch except root node.

Therefore, there are N-1 branches.

Therefore,

Therefore,

**Problem 4**

H1: Alpha uses channel hi

L1: Alpha uses channel lo

H2: Bravo uses channel hi

L2: Bravo uses channel lo

H3: Charlie uses channel hi

L3: Charlie uses channel lo

H4: Delta uses channel hi

L4: Delta uses channel lo

Through

There are:

Then have a Truth table：

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
| T | F | F | F | F | F |
| T | F | F | T | F | F |
| T | F | T | F | T | T |
| T | F | T | T | F | F |
| T | T | F | F | F | F |
| T | T | F | T | F | F |
| T | T | T | F | F | F |
| T | T | T | T | F | F |
| F | F | F | F | F | F |
| F | F | F | T | F | F |
| F | F | T | F | F | F |
| F | F | T | T | F | F |
| F | T | F | F | F | F |
| F | T | F | T | T | T |
| F | T | T | F | F | F |
| F | T | T | T | F | F |

1. There are two possible truth:

or

That fellow the rule that channels should be used interchangeably